PROBLEM 3-2

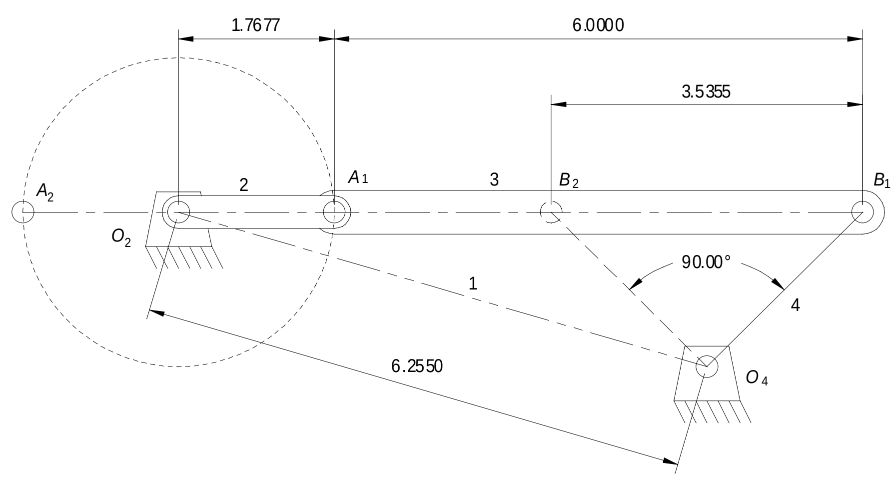
Statement: Design a fourbar Grashof crank-rocker for 90 deg of output rocker motion with no quick return. (See Example 3-1.) Build a cardboard model and determine the toggle positions and the minimum transmission angle.

Given: Output angle $\theta_4 := 90 \cdot deg$

Solution: See Example 3-1 and Mathcad file P0302.

Design choices: Link lengths: Link 3 $L_3 := 6.000$ Link 4 $L_4 := 2.500$

1. Draw the output link O_4B in both extreme positions, B_1 and B_2, in any convenient location such that the desired angle of motion θ_4 is subtended. In this solution, link 4 is drawn such that the two extreme positions each make an angle of 45 deg to the vertical.
2. Draw the chord B_1B_2 and extend it in any convenient direction. In this solution it was extended to the left.
3. Layout the distance A_1B_1 along extended line B_1B_2 equal to the length of link 3. Mark the point A_1.
4. Bisect the line segment B_1B_2 and layout the length of that radius from point A_1 along extended line B_1B_2. Mark the resulting point O_2 and draw a circle of radius O_2A_1 with center at O_2.
5. Label the other intersection of the circle and extended line B_1B_2, A_2.
6. Measure the length of the crank (link 2) as O_2A_1 or O_2A_2. From the graphical solution, $L_2 := 1.76775$
7. Measure the length of the ground link (link 1) as O_2O_4. From the graphical solution, $L_1 := 6.2550$



8. Find the Grashof condition.

$$
Condition(a,b,c,d) := \begin{array}{|l} S \leftarrow min(a,b,c,d) \\ L \leftarrow max(a,b,c,d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return\ \text{"Grashof"} \quad if\ \ SL < PQ \\ return\ \text{"Special Grashof"} \quad if\ \ SL = PQ \\ return\ \text{"non-Grashof"} \quad otherwise \end{array}
$$

$$
Condition(L_1, L_2, L_3, L_4) = \text{"Grashof"}
$$

PROBLEM 3-6

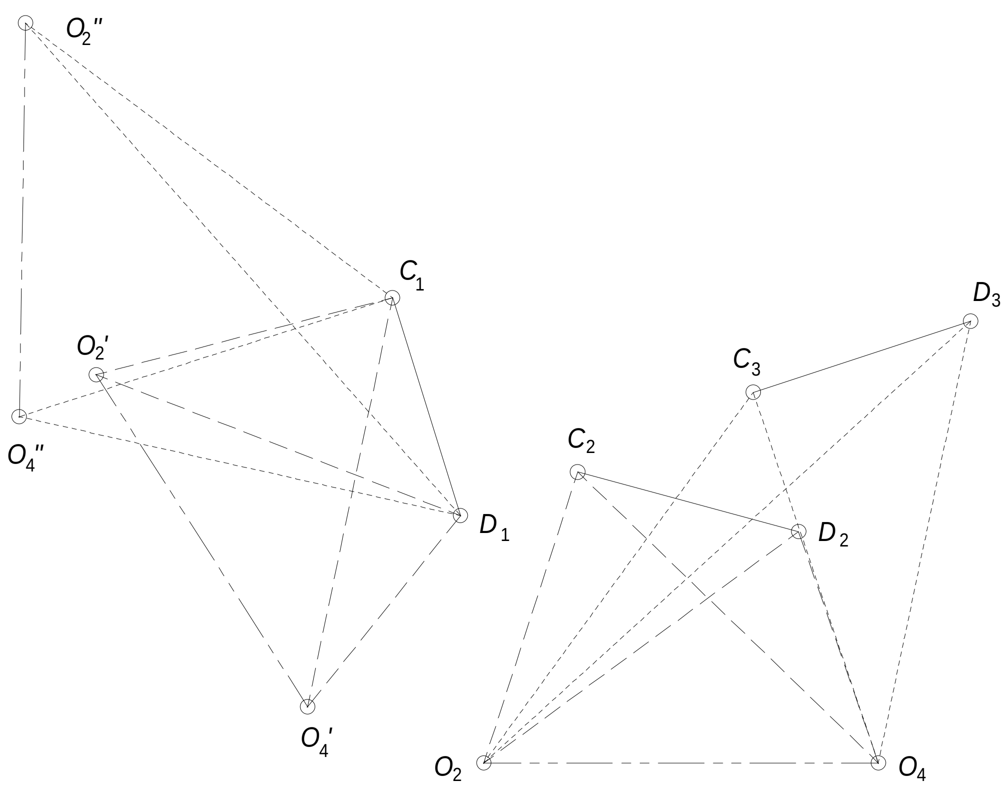
Statement: Design a fourbar mechanism to give the three positions shown in Figure P3-2 using the fixed pivots O_2 and O_4 shown. Build a cardboard model and determine the toggle positions and the minimum transmission angle. Add a driver dyad.

Solution: See Figure P3-2 and Mathcad file P0306.

Design choices:

Length of link 5: $L_5 := 5.000$ Length of link 2b: $L_{2b} := 2.000$

1. Draw link CD in its three design positions C_1D_1, C_2D_2, C_3D_3 in the plane as shown.
2. Draw the ground link O_2O_4 in its desired position in the plane with respect to the first coupler position C_1D_1.
3. Draw construction arcs from point C_2 to O_2 and from point D_2 to O_2 whose radii define the sides of triangle $C_2O_2D_2$. This defines the relationship of the fixed pivot O_2 to the coupler line CD in the second coupler position.
4. Draw construction arcs from point C_2 to O_4 and from point D_2 to O_4 whose radii define the sides of triangle $C_2O_4D_2$. This defines the relationship of the fixed pivot O_4 to the coupler line CD in the second coupler position.
5. Transfer this relationship back to the first coupler position C_1D_1 so that the ground plane position $O_2'O_4'$ bears the same relationship to C_1D_1 as O_2O_4 bore to the second coupler position C_2D_2.
6. Repeat the process for the third coupler position and transfer the third relative ground link position to the first, or reference, position.
7. The three inverted positions of the ground link that correspond to the three desired coupler positions are labeled O_2O_4, $O_2'O_4'$, and $O_2''O_4''$ in the first layout below and are renamed E_1F_1, E_2F_2, and E_3F_3, respectively, in the second layout, which is used to find the points G and H.



8. Draw construction lines from point E_1 to E_2 and from point E_2 to E_3.

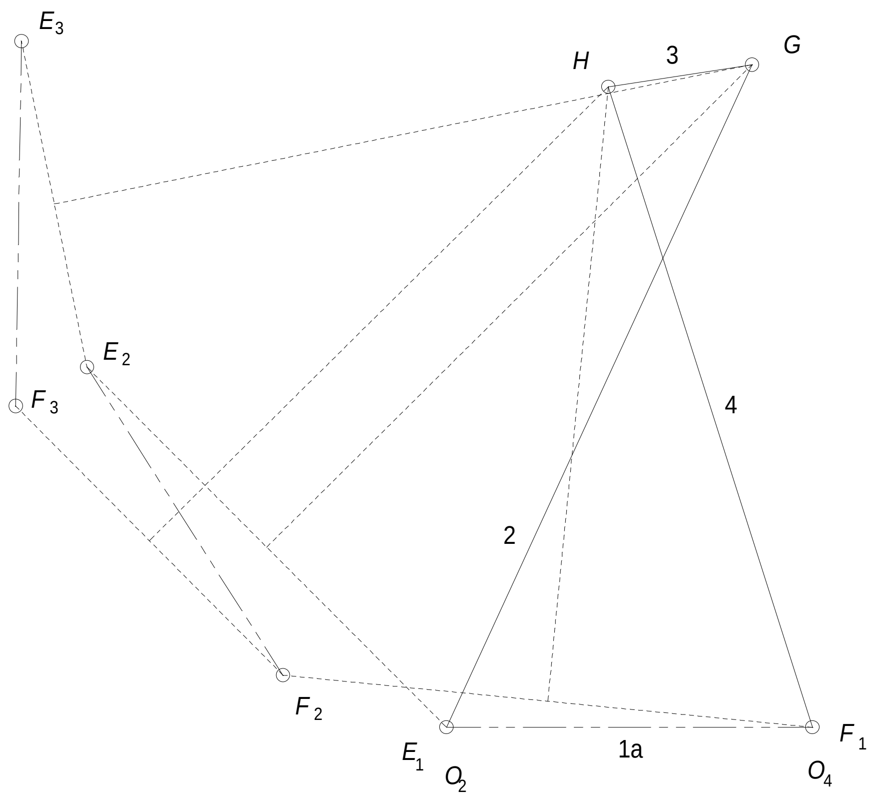
9. Bisect line E_1E_2 and line E_2E_3 and extend their perpendicular bisectors until they intersect. Label their intersection *G*.

10. Repeat steps 2 and 3 for lines F_1F_2 and F_2F_3. Label the intersection *H*.

11. Connect E_1 with *G* and label it link 2. Connect F_1 with *H* and label it link 4. Reinverting, E_1 and F_1 are the original fixed pivots O_2 and O_4, respectively.

12. Line *GH* is link 3. Line O_2O_4 is link 1a (ground link for the fourbar). The fourbar is now defined as O_2GHO_4 and has link lengths of

Ground link 1a	$L_{1a} := 4.303$	Link 2	$L_2 := 8.597$
Link 3	$L_3 := 1.711$	Link 4	$L_4 := 7.921$



13. Check the Grashof condition. Note that any Grashof condition is potentially acceptable in this case.

$$Condition(a, b, c, d) := \left| \begin{array}{l} S \leftarrow min(a, b, c, d) \\ L \leftarrow max(a, b, c, d) \\ SL \leftarrow S + L \\ PQ \leftarrow a + b + c + d - SL \\ return \ \text{"Grashof"} \quad if \ \ SL < PQ \\ return \ \text{"Special Grashof"} \quad if \ \ SL = PQ \\ return \ \text{"non-Grashof"} \quad otherwise \end{array} \right.$$

$$Condition\left(L_{1a}, L_2, L_3, L_4\right) = \text{"Grashof"}$$

The fourbar that will provide the desired motion is now defined as a Grashof double crank in the crossed configuration. It now remains to add the original points C_1 and D_1 to the coupler *GH* and to define the driving dyad.

14. Select a point on link 2 (O_2G) at a suitable distance from O_2 as the pivot point to which the driver dyad will be connected and label it B. (Note that link 2 is now a ternary link with nodes at O_2, B, and G.) In the solution below, the distance O_2B was selected to be $L_{2b} = 2.000$.

15. Draw a construction line through B_1B_3 and extend it up to the right.

16. Layout the length of link 5 (design choice) along the extended line. Label the other end A.

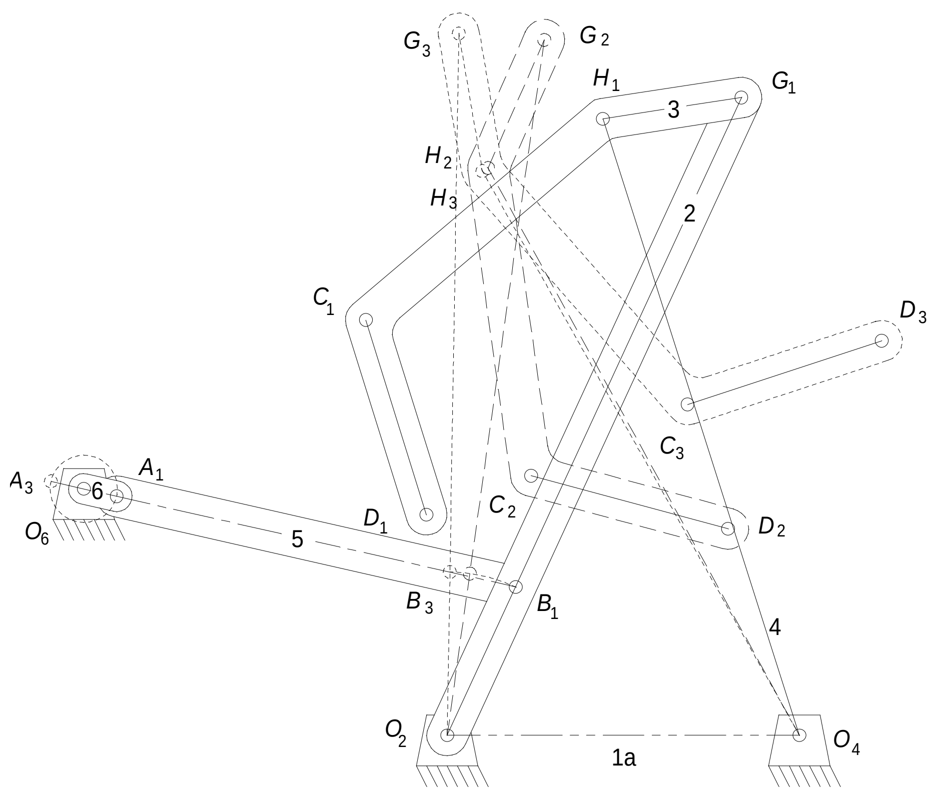
17. Draw a circle about O_6 with a radius of one-half the length B_1B_3 and label the intersections of the circle with the extended line as A_1 and A_3. In the solution below the radius was measured as $L_6 := 0.412$.

18. The driver fourbar is now defined as O_2BAO_6 with link lengths

Link 6 (crank)	$L_6 = 0.412$
Link 5 (coupler)	$L_5 = 5.000$
Link 1b (ground)	$L_{1b} := 5.369$
Link 2b (rocker)	$L_{2b} = 2.000$

19. Use the link lengths in step 18 to find the Grashoff condition of the driving fourbar (it must be Grashoff and the shortest link must be link 6).

$$Condition(L_6, L_{1b}, L_{2b}, L_5) = \text{"Grashof"}$$



PROBLEM 3-7

Statement: Repeat Problem 3-2 with a quick-return time ratio of 1:1.4. (See Example 3.9). Design a fourbar Grashof crank-rocker for 90 degrees of output rocker motion with a quick-return time ratio of 1:1.4.

Given: Time ratio $T_r := \frac{1}{1.4}$

Solution: See figure below for one possible solution. Also see Mathcad file P0307.

1. Determine the crank rotation angles α and β, and the construction angle δ from equations 3.1 and 3.2.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

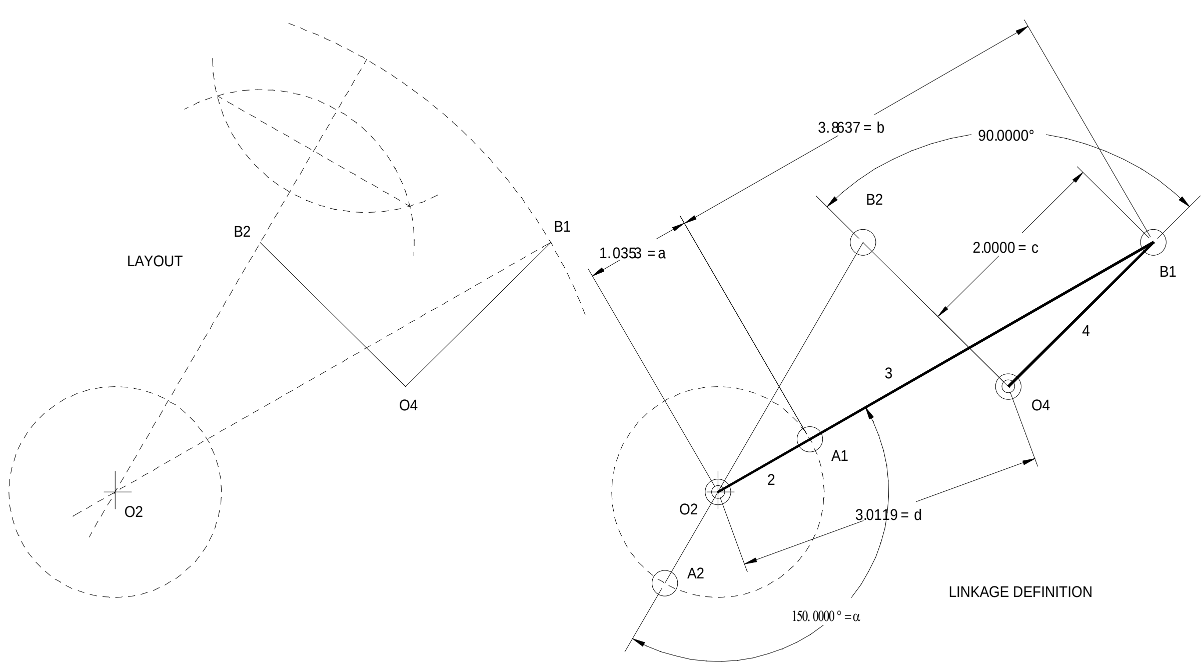
Solving for β, α, and δ

$$\beta := \frac{360 \cdot deg}{1 + T_r} \qquad \beta = 210\, deg$$

$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 150\, deg$$

$$\delta := \beta - 180 \cdot deg \qquad \delta = 30\, deg$$

2. Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 90 deg apart. Label one B_1 and the other B_2. In the solution below, each line makes an angle of 45 deg with the horizontal and has a length of 2.000 in.
3. Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below, the line is 30 deg to the horizontal.
4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (60 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2.
5. From O_2 draw an arc that goes through B_1. Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.



6. For this solution, the link lengths are:

Ground link (1)	$d := 3.0119 \cdot in$
Crank (2)	$a := 1.0353 \cdot in$
Coupler (3)	$b := 3.8637 \cdot in$
Rocker (4)	$c := 2.000 \cdot in$

PROBLEM 3-67

Statement: Design a fourbar Grashof crank-rocker for 120 degrees of output rocker motion with a quick-return time ratio of 1:2. (See Example 3-9.)

Given: Time ratio $T_r := \frac{1}{2}$

Solution: See figure below for one possible solution. Also see Mathcad file P0367.

1. Determine the crank rotation angles α and β, and the construction angle δ from equations 3.1 and 3.2.

$$T_r = \frac{\alpha}{\beta} \qquad \alpha + \beta = 360 \cdot deg$$

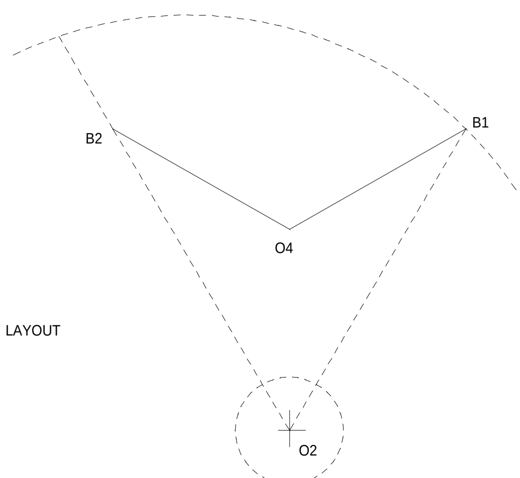
Solving for β, α, and δ

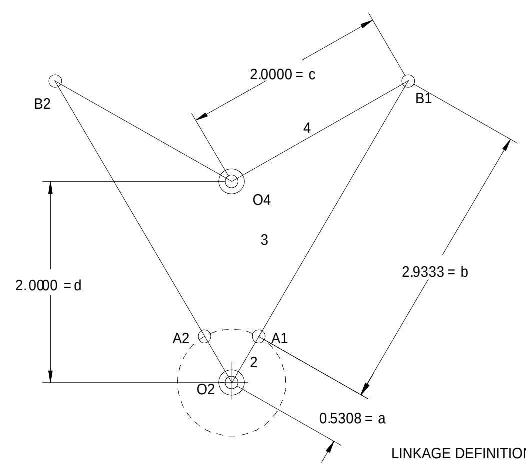
$$\beta := \frac{360 \cdot deg}{1 + T_r} \qquad \beta = 240\, deg$$

$$\alpha := 360 \cdot deg - \beta \qquad \alpha = 120\, deg$$

$$\delta := \beta - 180 \cdot deg \qquad \delta = 60\, deg$$

2. Start the layout by arbitrarily establishing the point O_4 and from it layoff two lines of equal length, 120 deg apart. Label one B_1 and the other B_2. In the solution below, each line makes an angle of 30 deg with the horizontal and has a length of 2.000 in.
3. Layoff a line through B_1 at an arbitrary angle (but not zero deg). In the solution below the line is 60 deg to the horizontal.
4. Layoff a line through B_2 that makes an angle δ with the line in step 3 (120 deg to the horizontal in this case). The intersection of these two lines establishes the point O_2.
5. From O_2 draw an arc that goes through B_1. Extend O_2B_2 to meet this arc. Erect a perpendicular bisector to the extended portion of the line and transfer one half of the line to O_2 as the length of the input crank.





6. For this solution, the link lengths are:

Ground link (1)	$d := 2.000 \cdot in$	Coupler (3)	$b := 2.9333 \cdot in$
Crank (2)	$a := 0.5308 \cdot in$	Rocker (4)	$c := 2.000 \cdot in$